Free expansion of a Bose-Einstein condensate in a 1D optical lattice

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We have experimentally investigated the free expansion of a Bose-Einstein condensate in an array of two-dimensional traps created by a one-dimensional optical lattice. If the condensate held in a magnetic trap is loaded adiabatically into the lattice, the increase in chemical potential due to the additional periodic potential is reflected in the expansion of the condensate after switching off the magnetic trap. We have calculated the chemical potential from measurements of the transverse expansion of the condensate as a function of the lattice parameters.

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The properties of Bose-Einstein condensates (BECs) in lower-dimensional trapping potentials have recently attracted increasing interest. Both theoretical and experimental investigations have been aimed at studying the differences between 1D [1–7] and 2D traps [8–14,4,6,7] with regard to the static and dynamic behaviour of BECs in such potentials. A thorough understanding of these properties is important as a prerequisite for predicting, e.g., the evolution of a BEC loaded into 1D and 2D waveguides. Condensates in 2D and 1D have been realized in magnetic traps starting from a 3D situation by changing the aspect ratio of the trap and the number of atoms in the condensate [4]. In order to realize a 1D condensate, optical dipole traps have been used to achieve the necessary asymmetry between the trapping frequencies [2,3]. Similarly, 2D condensates can be created in an array of pancake-shaped traps provided by the periodic potential of a 1D optical lattice [8,14]. For large lattice depths, i.e. in the tight-binding regime, the trapping frequencies along the lattice direction exceed those of the magnetic trap by orders of magnitude and the 1D lattice hence represents an array of 2D traps in which the motion along the direction of the array is frozen. Recently, Pedri *et al.* [13] have calculated the variation in chemical potential when the depth of the periodic potential is increased.

In this paper, we report on experiments with magnetically trapped BECs loaded into a one-dimensional optical lattice and subsequently allowed to expand freely inside the lattice. After the creation of an array of 2D traps within the 1D optical lattice, the magnetic trap is suddenly switched off. The condensate is then free to expand in the radial directions whilst still being confined in the lattice direction. The measured expansion allowed us to infer the chemical potential and to test its dependence on the lattice parameters.

Our experimental setup for creating Bose-Einstein condensates of rubidium atoms and loading these into 1D optical lattices has been described in detail elsewhere [15,16]. Briefly, we obtain BECs of $\approx 1-3\times10^4$ rubidium atoms inside a triaxial TOP-trap which are then loaded into a 1D optical lattice created by two independently controllable, linearly polarized Gaussian laser beams of wave-vector k and waist $w = 1.8\,\mathrm{mm}$, detuned by $25-40\,\mathrm{GHz}$ from the rubidium resonance line and propagating at an angle θ with respect to each other. The lattice constant d of the resulting one-dimensional periodic potential is then given by $d = \pi/(k\sin(\theta/2))$, and in the following we shall express the lattice depth U_0 in units of a re-scaled lattice recoil energy $E_{rec} = \hbar^2\pi^2/(2md^2)$ [16] corresponding to the respective lattice spacing d. In our setup we could realize both a horizontal lattice with $d = 1.56\,\mu\mathrm{m}$ and a vertical lattice with $d = 1.2\,\mu\mathrm{m}$. In the vertical case, after switching off the magnetic trap we accelerated the lattice downwards with an acceleration $a = 9.81\,\mathrm{m\,s^{-2}}$ by chirping the frequency difference between the lattice beams in order to compensate gravity in the rest frame of the lattice. Apart from the different lattice spacing, the vertical and horizontal cases were, therefore, equivalent.

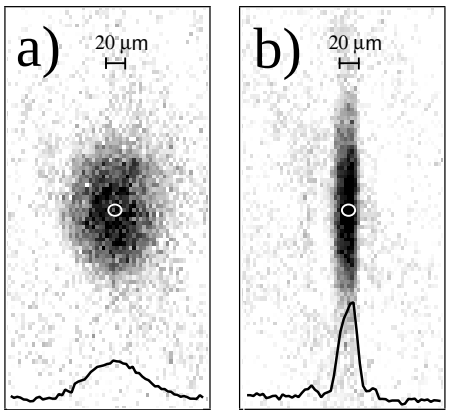

FIG. 1. Expanded condensate after 23 ms of time-of-flight without a lattice (a) and with a lattice of depth $U_0 = 20\,E_{rec}$ present during the expansion (b). The line plots represent the integrated profiles of the condensate density along the lattice direction; the reduced width $\rho_{||}$ along that direction (horizontal in this figure) is clearly visible. The white ellipses indicate the initial size of the condensate in the trap.

In order to load the condensate into the optical lattice, after adiabatically reducing the mean magnetic trap frequency $\overline{\omega}_{trap}$ to the desired value we linearly increased

the lattice depth from 0 to U_0 in a time $t_{ramp} = 150\,\mathrm{ms}$. Since typically the chemical potentials μ_0 of our condensates in traps with frequencies $\overline{\omega}_{trap}/2\pi \approx 20 - 70\,\mathrm{Hz}$ lie between 50 Hz and 200 Hz, the adiabaticity condition [17] $t_{ramp} > h/\mu_0$ was satisfied even for small chemical potentials. After that, *only* the magnetic trap was switched off and the condensate was imaged after a variable time t_{exp} of free expansion inside the optical lattice. The effect of a deep lattice on the free expansion is clearly evident in the condensate images of Fig. 1 and in the measurements of the condensate dimensions along ($\rho_{||}$) and perpendicular to the lattice direction ($\rho_\perp$) shown in Fig. 2 ($\rho_{||,\perp}$ denoting the e^{-1} half-width of a Gaussian fit to the density profile): Whilst in the lattice direction the condensate does not expand at all, the expansion in the direction perpendicular to the lattice is noticeably enhanced with respect to the expansion of the condensate in the absence of the optical lattice. The lack of expansion in the lattice direction reflects the fact that the condensate has effectively been split up into several smaller condensates confined in the individual lattice wells, whereas the enhanced expansion in the perpendicular direction is explained by the increase in the chemical potential when the lattice is ramped up.

Taking into account the periodic potential, Pedri *et al.* have calculated [13] that in the limit of large lattice depth ($U_0 \gg E_{rec}$) and assuming that the Thomas-Fermi approximation is always valid in the radial direction, the (local) chemical potential μ_k of well k is given by

$$\mu_k = \frac{1}{2} m \omega_{||}^2 d^2 (k_m^2 - k^2), \tag{1}$$

where $\omega_{||}$ is the magnetic trap frequency in the lattice direction (similarly, $\omega_\perp$ will indicate the trap frequency along the (observable) orthogonal direction). The well number k ranges from 0 to the maximum k_m given by

$$k_m = \left(\frac{2\hbar\overline{\omega}_{trap}}{m\omega_{||}^2 d^2} \right)^{1/2} \left(\frac{15}{8\sqrt{\pi}} N \frac{a_s}{a_{ho}} \frac{d}{\sigma_{||}} \right)^{1/5}, \tag{2}$$

where N is the total number of condensate atoms, $a_{ho} = \sqrt{\hbar/(m\overline{\omega})}$ the harmonic oscillator length, a_s is the s-wave scattering length and $\sigma_{||}$ is the width along the lattice direction of a Gaussian wavepacket in a single lattice well. We calculated $\sigma_{||}$ by using a variational ansatz with a Gaussian wavefunction [18]. The total number of wells is given by $n = 2k_m + 1$. For the central well with $k = 0$, the (local) chemical potential is

$$\mu_{k=0} = \left(\frac{\pi}{2} \right)^{1/5} \left(\frac{U_0}{E_{rec}} \right)^{1/10} \mu_0, \tag{3}$$

where μ_0 is the chemical potential in the absence of the lattice. We have checked that considering only the central well for the perpendicular width reproduces to within a small error ($< 1\%$) the result of a fit to the envelope of all lattice wells.

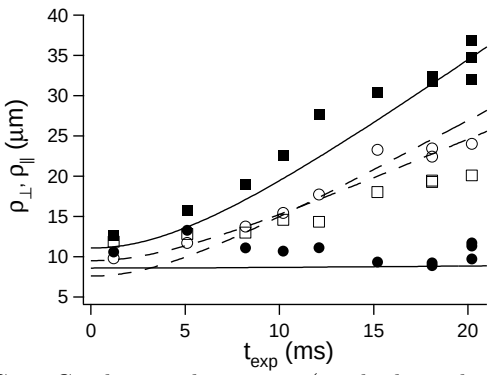

FIG. 2. Condensate dimensions (single shot values) $\rho_{||}$ (circles) and $\rho_\perp$ (squares), versus the expansion time t_{exp} in the presence of a horizontal lattice of depth $U_0 \approx 25\,E_{rec}$ (filled symbols) and without lattice (open symbols). The condensate was initially released from a trap with $\overline{\omega}_{trap}/2\pi = 25\,\mathrm{Hz}$ ($\omega_\perp/\omega_{||} = 1/\sqrt{2}$). The dashed and solid lines are the results of a numerical simulation (see text).

We modeled the expansion of the condensate in the presence of the 1D lattice using the equations derived in [19,20] with a slight modification along the lines of Ref. [7]: In the ansatz leading to the differential equation of [19] for the scaling factor $\lambda_{||}(t = t_{exp}) = \rho_{||}(t = t_{exp})/\rho_{||}(t = 0)$ along the lattice direction we replaced the atomic mass m by the effective mass $m^*(U_0)$, introduced in [7] in analogy with a solid state physics approach, as derived from a band structure calculation for a periodic potential of depth U_0. As can be seen in Figs. 2 and 3, taking into account the variation of $\mu_{k=0}$ with U_0 this model reproduces well our experimental data for the perpendicular expansion of the condensate (the theoretical plots are corrected for the $5\,\mu$m resolution of our imaging system). For the lattice direction, this approach gives the correct result for deep lattices ($U_0 \gg E_{rec}$) and reproduces reasonably well the qualitative behaviour in the intermediate regime ($0 < U_0 < 5\,E_{rec}$). We find, however, that the experimental expansion along the lattice direction is considerably less than the theoretical prediction. We have checked that taking into account the finite momentum spread of the condensate when calculating the effective mass only leads to a correction on the percent level and thus cannot explain the deviation of our experimental data from the numerical calculations neglecting mean field corrections. This might indicate that there are effects such as self-trapping [21] due to the mean-field interaction that further reduce the expansion in the lattice direction.

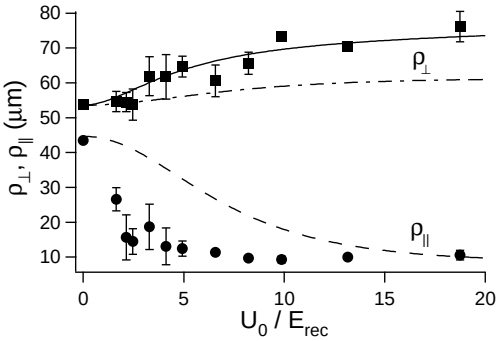

FIG. 3. Dependence on the lattice depth of the size of a condensate released from a trap with $\overline{\omega}_{trap}/2\pi = 25\,\mathrm{Hz}$ ($\omega_\perp/\omega_{||} = \sqrt{2}/1$) after a time-of-flight of 22 ms for a vertical lattice. For $\rho_\perp$ the solid line is the result of the numerical integration of the scaling equations (see text) taking into account the variation of the chemical potential $\mu_{k=0}$ with the lattice depth, whereas the dot-dashed line was calculated assuming a fixed μ_0. In the lattice direction (circles), the experimentally measured expansion for $\rho_{||}$ lies below the theoretical prediction. The error bars indicate the standard deviations of the averaged data points.

For the perpendicular direction we have included in Fig. 3 the results of a numerical simulation *without* taking into account the variation of the chemical potential with the lattice depth, *i.e* using μ_0 instead of $\mu_{k=0}$ of Eqn. 4. Clearly, the resulting increase in perpendicular size is much less than what we observe experimentally and is related to the different re-distribution of mean-field energy when the degree of freedom in the lattice direction is frozen out. We have checked that the variation in lattice depth during the time-of-flight of up to 23 ms in the Gaussian beam profile (during which time the condensate drops by up to 2.6 mm) does not affect the final widths. In fact, the mean-field explosion (during which mean-field energy is converted into kinetic energy) occurs during the first few milliseconds (equivalent to $< 100\,\mu$m of free fall), after which the confinement along the lattice direction is almost perfect as long as the local lattice depth is larger than a few recoil energies.

We could also directly deduce from our data the dependence of the chemical potential on U_0 by calculating $\mu_{k=0}$ from the initial perpendicular size $\rho_\perp(t=0)$ of the condensate in the presence of the lattice inferred from the size $\rho_\perp(t=t_{exp})$ measured after an expansion time t_{exp} and the ratio $\lambda_\perp(t=t_{exp})/\lambda_\perp(t=0)$ of the scale factor $\lambda_\perp(t)$. The chemical potential was then given by

$$\mu_{k=0} = \frac{1}{2} m\omega_\perp^2 \rho_\perp^2(t=0). \qquad (4)$$

In Fig. 4 we plot the ratio $\mu_{k=0}/\mu_0$ between the chemical potential as measured from the expansion of the condensate in the optical lattice and the chemical potential μ_0 in the magnetic trap alone. Agreement with Eqn. 3 is very good.

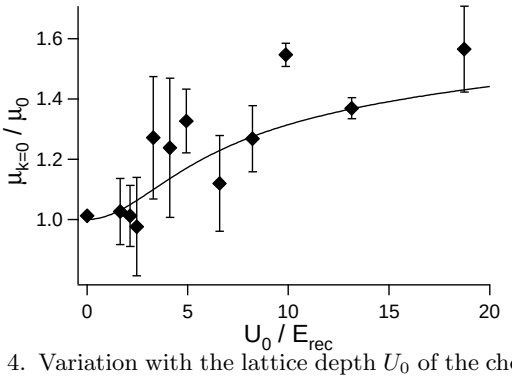

FIG. 4. Variation with the lattice depth U_0 of the chemical potential $\mu_{k=0}$ (in units of μ_0 in the absence of the lattice) calculated from the data of Fig. 3. The solid line is the theoretical prediction of Pedri *et al.*

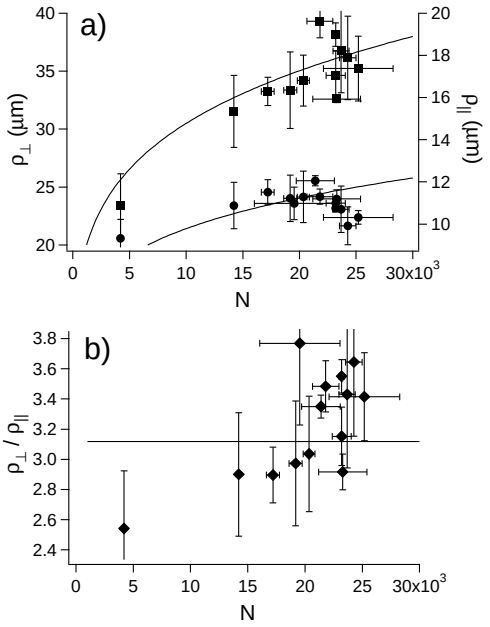

FIG. 5. Variation of the condensate widths (a) along ($\rho_{||}$, circles) and perpendicular to the lattice direction ($\rho_\perp$, squares), and of the the aspect ratio (b) with the number of atoms N. The solid lines are predictions based on Eqn. 4 for the chemical potential $\mu_{k=0}$ in the presence of the lattice. In this experiment $U_0/E_{rec} = 20$, $\overline{\omega}_{trap}/2\pi = 12\,\mathrm{Hz}$ with $\omega_\perp/\omega_{||} = \sqrt{2}/1$. The error bars represent standard deviations for the averaged data points at fixed final RF cut.

We have also checked the dependence of $\mu_{k=0}$ on the number of atoms N in the condensate. As N is varied by cutting into the (almost pure) condensate during the final stage of the RF evaporation, for a deep lattice the size of the condensate after a time-of-flight increases considerably in the perpendicular direction when N increases, as expected from the variation in chemical potential (see Fig. 5 (a)). In the lattice direction, however, the increase in size shows a weaker dependence on N than expected.

For $N > 2 \times 10^4$, $\rho_{||}$ even starts to decrease with increasing condensate number. Consequently, the aspect ratio $\rho_\perp/\rho_{||}$ increases with N (see Fig. 5) rather than remaining constant as theoretically predicted. This behaviour suggests that the condensate width in the lattice direction does not fully reflect the change in chemical potential. When N gets large, mean-field effects might become important in the dynamics of the condensate expansion, as mentioned above, and non-linear effects like self-trapping might reduce the observed width $\rho_{||}$.

Finally, we note here that in the limit of large lattice depths, our experiments effectively realize an adiabatic transformation between a 3D condensate and an array of 2D condensates. The condition $\mu_{3D} < \hbar\omega_{lat}$ of Ref. [4] (where $\omega_{lat} = 2(E_{rec}/\hbar)\sqrt{U_0/E_{rec}}$ is the harmonic approximation for the oscillation frequency in a lattice well) for the condensates in each well to be in the 2D limit is always satisfied for the small number of atoms in a single well ($\approx 10^3$) present in our experiment. For an array of 2D condensates obtained by creating the condensate in the combined potential of the harmonic trap and the lattice, Burger *et al.* [14] have shown that in the case of their cigar-shaped condensate (with the long axis along the lattice direction), the transition temperature T_c^{2D} in the presence of the lattice is significantly lower than T_c^{3D} in the 3D case (i.e. in the magnetic trap without the lattice). Calculating the critical temperature T_c^{2D} along the same lines for our system, we find that $T_c^{2D} \approx T_c^{3D}$ due to the larger number of atoms per lattice site in our geometry, and hence we expect no significant change in the condensate fraction in the presence of the lattice. In fact, experimentally we even find a consistently larger condensate fraction after ramping up the lattice. This result indicates that, with an appropriate choice of parameters, a 1D optical lattice could be used to investigate adiabatic transformations between 3D and 2D condensates which could, e.g., be exploited to create condensates from thermal clouds by changing the dimensionality of the system, similarly to the change in the shape of the potentials in Refs [22,23] for other geometries.

In summary, we have studied the free expansion of a Bose-Einstein condensate in a one-dimensional optical lattice. When increasing the lattice depth, an increase in the chemical potential of the condensate was measured, in agreement with a recent theoretical calculation [13]. The expansion along the lattice direction, though, seems to be strongly reduced in the regime of intermediate lattice depth, necessitating a more thorough theoretical treatment taking into account mean-field effects in the dynamics after the magnetic trap is switched off. Furthermore, the possibility of adiabatically changing the dimensionality between a 3D trap and an array of 2D traps should, with appropriate experimental parameters, represent a way to reach quantum degeneracy by adding a periodic potential to a harmonic trap. The improved characterization of the behaviour of condensates in optical lattices is important with regard to adiabaticity criteria when a condensate is transferred into a periodic potential that could, e.g., be used to implement a neutral atom quantum computer.

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